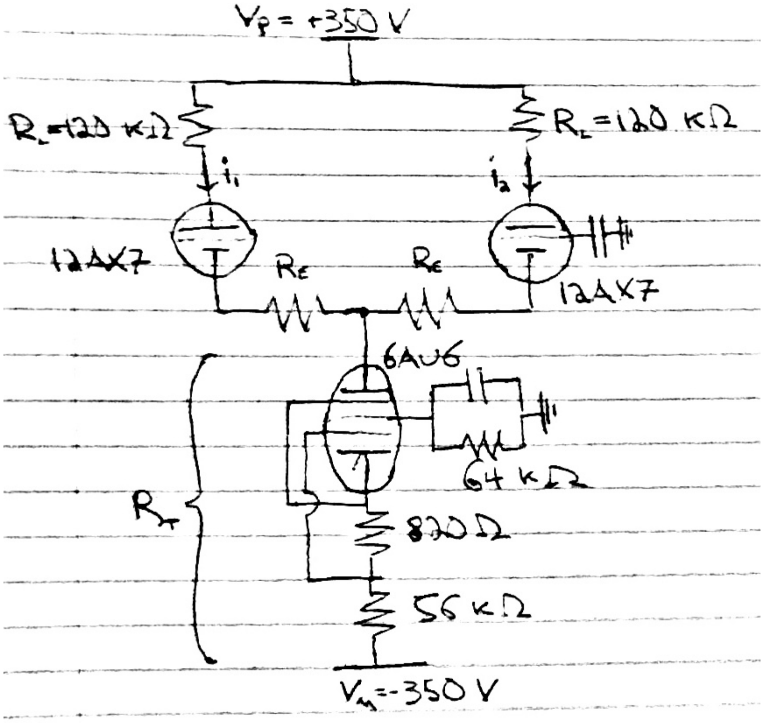

Figure 1: Differential amplifier made with a 12AX7 twin triode and a 6AU6 sharp-cutoff pentode. The signal gets applied to the grid of the triode on the left-hand side (≡ side 1). The capacitor on the grid of the triode on the right-hand side (≡ side 2) pins the grid at ac ground.

The voltage across a triode $V_a(V_g, i)$ is a function of the grid-cathode voltage V_g and the current through the tube i. From chain rule,

$$\begin{aligned}\frac{\mathrm{d}V_a}{\mathrm{d}V_g} &= \left.\frac{\partial V_a}{\partial V_g}\right|_{i=\text{const.}} + \left.\frac{\partial V_a}{\partial i}\right|_{V_g=\text{const.}} \frac{\mathrm{d}i}{\mathrm{d}V_g} \\ &= -\mu + r_a \frac{\mathrm{d}i}{\mathrm{d}V_g},\end{aligned} \tag{1}$$

where μ is the amplification factor of the tube and r_a is the anode resistance. In practice, the relevant gain of the circuit is relative to the voltage V_G applied to the grid with respect to ground. With $V_g = V_G - V_C$ and the cathode voltage $V_C = V_P - R_L i - V_a$,

$$\frac{\mathrm{d}V_g}{\mathrm{d}V_G} = 1 + R_L \frac{\mathrm{d}i}{\mathrm{d}V_G} + \frac{\mathrm{d}V_a}{\mathrm{d}V_G}. \tag{2}$$

With $\mathrm{d}V_a/\mathrm{d}V_G = (\mathrm{d}V_a/\mathrm{d}V_g)(\mathrm{d}V_g/\mathrm{d}V_G)$ and $(\mathrm{d}i/\mathrm{d}V_g)(\mathrm{d}V_g/\mathrm{d}V_G) = (\mathrm{d}i/\mathrm{d}V_G)$, the combination of eqs. 1 and 2 yields

$$\frac{\mathrm{d}V_a}{\mathrm{d}V_G} = -\mu\left(1 + \frac{\mathrm{d}V_a}{\mathrm{d}V_G}\right) + \frac{\mathrm{d}i}{\mathrm{d}V_G}\left(-\mu R_L + r_a\right). \tag{3}$$

From an additional application of chain rule,

$$\frac{\mathrm{d}V_a}{\mathrm{d}V_G} = -\mu\left(1 + \frac{\mathrm{d}V_a}{\mathrm{d}V_G}\right) + \frac{\mathrm{d}i}{\mathrm{d}V_a}\frac{\mathrm{d}V_a}{\mathrm{d}V_G}\left(-\mu R_L + r_a\right), \tag{4}$$

and solving for $\mathrm{d}V_a/\mathrm{d}V_G$ results in

$$\frac{\mathrm{d}V_a}{\mathrm{d}V_G} = -\left[1 + \frac{1}{\mu} + \frac{\mathrm{d}i}{\mathrm{d}V_a}(R_L - \frac{r_a}{\mu})\right]^{-1}. \tag{5}$$

Finally, the output voltage $V_L = -R_L i$ is given by

$$\frac{\mathrm{d}V_L}{\mathrm{d}V_G} = -R_L \frac{\mathrm{d}i}{\mathrm{d}V_a}\frac{\mathrm{d}V_a}{\mathrm{d}V_G} = R_L\left[\left(\frac{\mathrm{d}i}{\mathrm{d}V_a}\right)^{-1}\left(1 + \frac{1}{\mu}\right) + \left(R_L - \frac{r_a}{\mu}\right)\right]^{-1}. \tag{6}$$

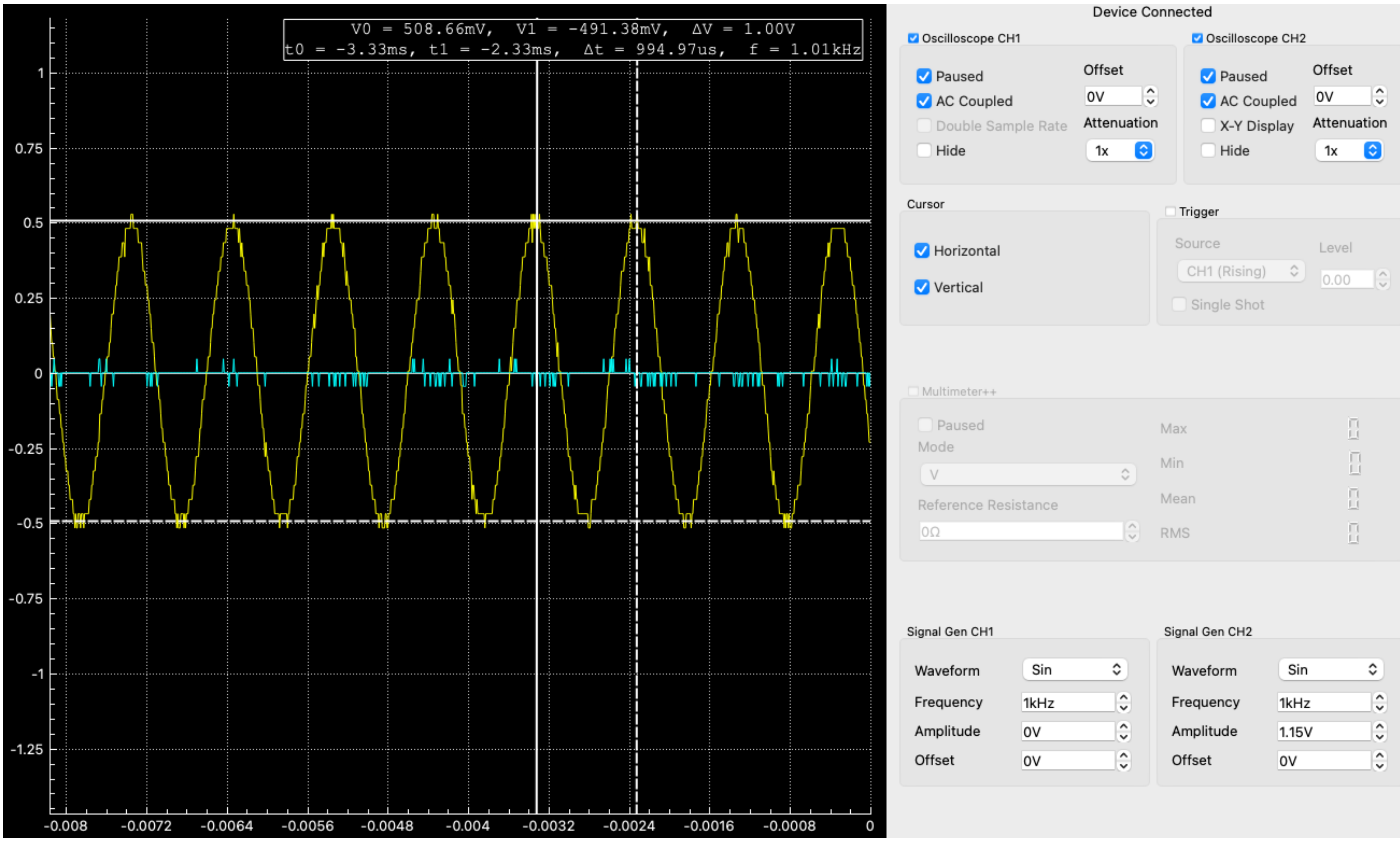

Figure 2: Input signal applied to one of the triode grids; 1 kHz, 1 V peak-to-peak.

So, solving for the gain has boiled down to determining $\mathrm{d}i/\mathrm{d}V_a$.

The necessary equations describing the voltages in the circuit are

$$V_{a1} + i_1(R_L + R_E) - V_{a2} - i_2(R_L + R_E) = 0 \tag{7}$$

$$V_{a1} + i_1(R_L + R_E) + (i_1 + i_2)R_T = V_P - V_M \tag{8}$$

$$-V_{g2} = V_P - V_{a1} - i_1(R_L + R_E) + i_2 R_E \tag{9}$$

For eq. 9, it has been assumed that the capacitor on the grid of tube 2 places the grid at ac ground, so the grid-cathode voltage V_{g2} is equal and opposite to the cathode voltage.

Differentiating eq. 7 with respect to V_{a1}:

$$1 + \frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}}(R_L + R_E) - \frac{\mathrm{d}V_{a2}}{\mathrm{d}V_{g2}}\frac{\mathrm{d}V_{g2}}{\mathrm{d}V_{a1}} - \frac{\mathrm{d}i_2}{\mathrm{d}V_{a1}}(R_L + R_E) = 0 \tag{10}$$

$$1 + \frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}}(R_L + R_E) - (-\mu + r_{a2}\frac{\mathrm{d}i_2}{\mathrm{d}V_{g2}})\frac{\mathrm{d}V_{g2}}{\mathrm{d}V_{a1}} - \frac{\mathrm{d}i_2}{\mathrm{d}V_{a1}}(R_L + R_E) = 0 \tag{11}$$

$$1 + \frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}}(R_L + R_E) + \mu\frac{\mathrm{d}V_{g2}}{\mathrm{d}V_{a1}} - r_{a2}\frac{\mathrm{d}i_2}{\mathrm{d}V_{a1}} - \frac{\mathrm{d}i_2}{\mathrm{d}V_{a1}}(R_L + R_E) = 0 \tag{12}$$

Differentiating eq. 8 with respect to V_{a1}:

$$1 + \frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}}(R_L + R_E) + \left(\frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}} + \frac{\mathrm{d}i_2}{\mathrm{d}V_{a1}}\right)R_T = 0 \tag{13}$$

$$\rightarrow \frac{\mathrm{d}i_2}{\mathrm{d}V_{a1}} = -\frac{1 + (\mathrm{d}i_1/\mathrm{d}V_{a1})\left[R_L + R_E + R_T\right]}{R_T} \tag{14}$$

Differentiating eq. 9 with respect to V_{a1}:

$$-\frac{\mathrm{d}V_{g2}}{\mathrm{d}V_{a1}} = -1 - \frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}}(R_L + R_E) + \frac{\mathrm{d}i_2}{\mathrm{d}V_{a1}}R_E \tag{15}$$

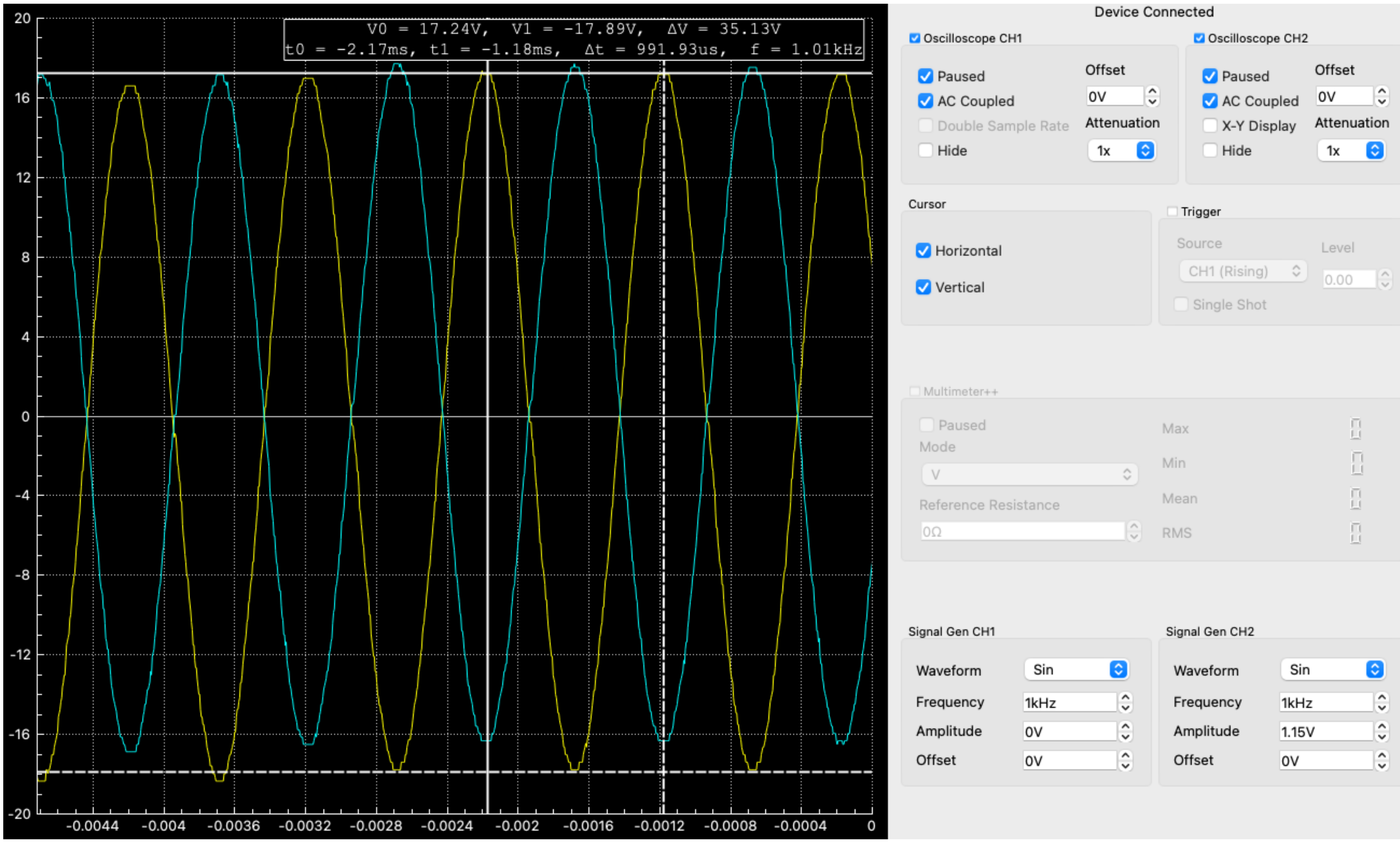

Figure 3: For the input signal shown in fig. 2, the output signals on, in yellow, the same side of the diff amp as that to which the input is applied and, in blue, the opposite side. As expected, the outputs are 180° out-of-phase. They appear to have different amplitudes, but that is because the oscilloscope is not calibrated correctly. When the output signals are measured individually on the channel corresponding to the yellow trace, they are both seen to have a peak-to-peak voltage of 35 V.

By combining eqs. 12, 15, and 14,

$$\frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}} = -\left[R_T + \frac{r_{a2}+R_L}{1+\mu} + R_E\right] \Big/ \left[(R_L+R_E)R_T + (R_L+R_E+R_T)\left(\frac{r_{a2}+R_L}{1+\mu} + R_E\right)\right]. \tag{16}$$

For the ac resistance $R_T = 1$ MΩ of the 6AU6 sharp-cutoff pentode, which is much larger than all the other resistances,

$$\frac{\mathrm{d}i_1}{\mathrm{d}V_{a1}} \approx -\left[(R_L+2R_E) + \frac{r_{a2}+R_L}{1+\mu}\right]^{-1}. \tag{17}$$

This can be inserted into eq. 6 to get an expression for the gain on the side of the diff amp to which the signal is applied:

$$\lim_{R_T\to\infty} \frac{\mathrm{d}V_L}{\mathrm{d}V_G} = -\frac{\mu R_L/2}{R_L + r_a + R_E(1+\mu)}. \tag{18}$$

Here, both sides of the twin triode tube have been taken to have anode resistance r_a. This answer – the gain is half of what it would be for the analogous common cathode amplifier – is obvious in retrospect for $R_E = 0$. With $R_T \to \infty$, the two sides must be completely balanced, which means the triodes must see equal signals. For this to happen, the ac voltage on the cathodes has to rise to half the input side ac grid voltage, which drives the triode on the opposite side while lowering the grid-cathode voltage on the input side.

For the diff amp in fig. 1, the operating point of the triodes is at a plate-cathode voltage of $V_a = 244$ V and plate current $i = 1.02$ mA. According to the 12AX7 datasheet, the anode resistance is $r_a = 64$ kΩ and the amplification factor is $\mu = 100$. Plugging in these numbers into eq. 18 along with $R_L = 120$ kΩ and $R_E = 0$ kΩ results in a theoretical gain of 32.6. This is approximately seen in practice for the 1 kHz, 1 V peak-to-peak grid signal shown in fig. 2, which gets amplified to the 35 V peak-to-peak signal shown in fig. 3.

With $R_E = 0$, for inputs on the grid of side 1, the ratio of the side 1 and side 2 gains is

$$\frac{\mathrm{d}i_2}{\mathrm{d}i_1} = -1 + \frac{1}{R_T}\left(\frac{r_a+R_L}{1+\mu}\right), \tag{19}$$

where the expressions from eqs. 14 and 16 have been used. The second term is small for any reasonable choice of resistors, so the outputs are out-of-phase. Even for a modest tail resistance like 100 kΩ, their amplitudes are the same to within a few percent. For the pentode that I used with an ac resistance of $R_T = 1$ MΩ, the amplitudes are theoretically essentially equal. This is seen in practice, but my oscilloscope is not calibrated well enough for it to be demonstrated in fig. 3.