

PUSH-PULL CIRCUIT ANALYSIS*

Cathode-Coupled Output Stage

By S. W. Amos, B.Sc.(Hons.), Grad.I.E.E.

(B.B.C. Engineering Training Department)

MOST circuits used for providing the out-of-phase signals necessary for driving push-pull stages of amplification employ a valve or centre-tapped transformer to perform the "phase-splitting" but the circuit given in Fig. 1 requires neither of these components. This circuit is one in which one output valve derives its input by cathode-coupling from the other. In its application to resistance-coupled intermediate A.F. stages it is quite well-known^{1,2} but its suitability for stages having a transformer output coupling is less widely recognised. It has, however, been used in this way by C. G. Mayo and H. D. Ellis in the design of certain B.B.C. audio-frequency amplifiers.

The input signal is applied to the terminals AB , of which B is earthed, and the two valves V_1 and V_2 (usually high-slope tetrodes or pentodes, but drawn as triodes for convenience) operate automatically in push-pull. The phase-reversal is accomplished as follows. Consider the circuit with V_2 and all its grid circuit components removed. The valve V_1 then operates normally but with some current feedback due to the A.F. potentials developed across the unbypassed grid bias resistance R_k and the resistance R_0 . These A.F. potentials provide the input signal for V_2 when it is in circuit.

In order that the A.C. component of the anode current of V_2 shall be 180 deg. out of phase with that of V_1 , the cathode of V_2 must be joined to the cathode of V_1 and its grid must be joined to earth. This latter connection is actually made via a capacitor in order that the correct value of grid bias may be applied to V_2 by means of the grid leak R_2 . Thus the anode currents of both valves pass through R_k (which hence acts as a common grid bias resistance) and R_0 and the amplitudes of the A.C. components of these anode currents adjust themselves so that they are unequal, V_1 providing the greater A.C. component. This difference in amplitude is necessary in order to provide the input signal for V_2 . It is the purpose of

this analysis to find the effect of varying R_0 on the performance of the circuit and hence to find an expression for its optimum value.

Let i_1 and i_2 = amplitude of the A.C. components of the anode currents of V_1 and V_2 respectively (both currents are regarded as positive, their directions being indicated in Fig. 1, and i_1 is assumed greater than i_2).

E_1 and E_2 = amplitudes of the alternating potentials between grid and cathode of V_1 and V_2 respectively

E_k = amplitude of the P.D. developed across R_k

E_0 = amplitude of the P.D. developed across R_0

E = amplitude of the input signal applied across AB .

From Fig. 1, we have

$$\begin{aligned} E_1 &= E - (E_0 + E_k) = \\ &= E - [(i_1 - i_2)R_0 + (i_1 - i_2)R_k] \\ &= E - (i_1 - i_2)(R_0 + R_k) \\ &= E - (i_1 - i_2)R_0' \quad \dots \quad (1) \end{aligned}$$

in which $R_0' = R_0 + R_k$

$$\begin{aligned} \text{Also } E_2 &= E_0 + E_k \\ &= (i_1 - i_2)(R_0 + R_k) \\ &= (i_1 - i_2)R_0' \quad \dots \quad (2) \end{aligned}$$

Regarding V_1 and V_2 as constant current generators, we have

$$i_1 = g_m E_1 = g_m [E - (i_1 - i_2)R_0'] \quad (3)$$

and

$$i_2 = g_m E_2 = g_m (i_1 - i_2)R_0' \quad \dots \quad (4)$$

in which V_1 and V_2 are assumed to be valves with the same value of g_m . It is considered unnecessary in equations (3) and (4) to include negative signs, thus, $i_1 = -g_m E_1$. With the input signals and anode currents of V_1 and V_2 as indicated in Fig. 1, all phase relationships are taken care of by using positive signs throughout as in equations (3) and (4).

$$\text{From (3)} \quad (i_1 - i_2)R_0' = E - \frac{i_1}{g_m}$$

* MS. accepted by the Editor, September 1945.

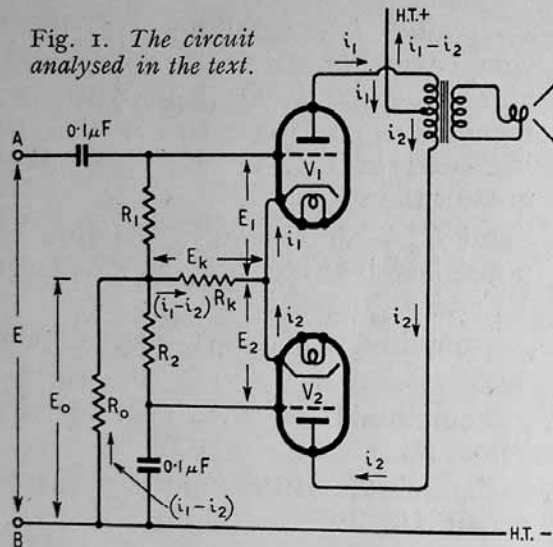
$$\text{From (4)} \quad (i_1 - i_2)R_0' = \frac{i_2}{g_m}$$

$$\therefore \quad E - \frac{i_1}{g_m} = \frac{i_2}{g_m}$$

$$i_1 + i_2 = g_m E$$

which shows that the combined output currents of V_1 and V_2 have the same value that would be given by a single valve with the same value of g_m and the same total input E .

Fig. 1. The circuit analysed in the text.



Solving (3) and (4) for i_1 and i_2 we find

$$i_1 = \frac{(1 + g_m R_0') g_m E}{1 + 2g_m R_0'} \quad \dots \quad (5)$$

$$\text{and } i_2 = \frac{g_m^2 R_0' E}{1 + 2g_m R_0'} \quad \dots \quad (6)$$

$$\therefore i_1 - i_2 = \frac{g_m E}{1 + 2g_m R_0'} \quad \dots \quad (7)$$

In order to assess the difference between i_1 and i_2 we shall use the symbol Δ to represent (difference in amplitude of the A.C. components of the anode currents of V_1 and V_2) / (mean amplitude of the A.C. components)

$$\text{i.e. } \Delta = \frac{i_1 - i_2}{\frac{1}{2}(i_1 + i_2)}$$

From the expressions given above for $i_1 - i_2$ and $i_1 + i_2$

$$\Delta = \frac{2g_m E}{1 + 2g_m R_0'} \cdot \frac{1}{g_m E} = \frac{2}{1 + 2g_m R_0'}$$

In most practical problems it will be found that $2g_m R_0'$ considerably exceeds unity, so that

$$\Delta \approx \frac{1}{g_m R_0'}$$

$$\therefore R_0' \approx \frac{1}{\Delta g_m} \quad \dots \quad (8)$$

Suppose, for a particular type of valve, $g_m = 10$ mA/volt and we do not wish the difference in A.C. components of the anode currents to exceed 5 per cent. of their mean value. We have, from (8).

$$R_0' \approx \frac{1}{\frac{1}{20} \cdot \frac{10}{1,000}} = 2,000 \text{ ohms.}$$

R_0' is, of course, the value of total cathode resistance, i.e., grid bias resistance R_k , together with the coupling resistance R_0 . If R_k is 100 ohms then R_0 will need to be 1,900 ohms. We shall deal throughout this manuscript, however, with the value of R_0' rather than R_0 . A value of R_0' of 2,000 ohms may be considered excessive, since the D.C. components of the anode currents of V_1 and V_2 which pass through it may cause the safe heater-cathode insulation rating of the valves to be exceeded. If the total cathode current of each valve is 40 mA and if R_0' is 2,000 ohms, then the steady P.D. across R_0' is $2,000 \times \frac{80}{1,000} = 160$ volts. R_0' will, therefore, need to be capable of dissipating steadily a power of $\frac{80}{1,000} \times 160 = 12.8$ watts, and if the valves require an anode potential of 250 volts,

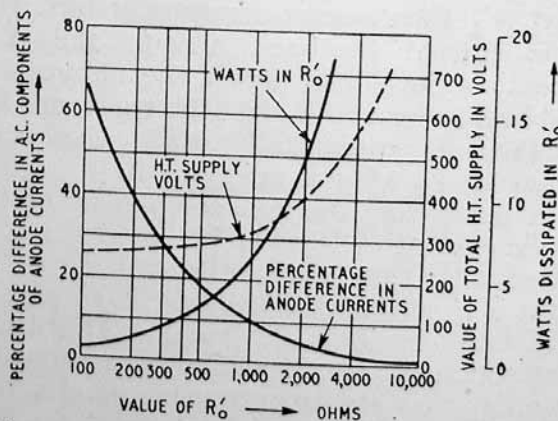


Fig. 2. Curves illustrating the variation of H.T. supply volts, the percentage difference in anode currents and the power in R_0' with variation of R_0' . These results apply to valves with $g_m = 10$ mA/volt and total cathode current = 40 mA per valve.

the total H.T. voltage necessary will be $250 + 160 = 410$, which may be considered excessive. The dependence of Δ , power in R_0' and H.T. voltage on the value of R_0' are illustrated in Fig. 2, evaluated for

valves with $g_m = 10$ mA/volt and total cathode current equal to 40 mA per valve. From this a most suitable value for R_0' would appear to be about 1,000 ohms. It would be good enough, probably, to make $R_0 = 1,000$ ohms (a standard value), R_k being 100 ohms (a typical value) so that $R_0' = 1,100$ ohms. Clearly, for a given value of Δ , in order that the power wasted

$g_m = 10$ mA/volt and $R_0' = 1,000$ ohms as before. We have

$$\frac{E}{E_1} = \frac{1 + 2 \times \frac{10}{1,000} \times 1,000}{1 + \frac{10}{1,000} \times 1,000} = \frac{21}{11} \approx 1.91$$

so that the input signal required by the circuit is, in these circumstances, 1.91 times that required by one of the push-pull valves. Since i_1 is greater than i_2 , it follows that V_1 has a larger input signal than V_2 and so overloads first. Suppose V_1 can accept a maximum input signal of 8 volts peak value. The input necessary to the circuit is $8 \times 1.91 = 15.28$ volts for maximum output power.

The dependence of $\frac{E}{E_1}$ on the value of $g_m R_0'$ is illustrated in Fig. 3, from which it can be seen that the greater $g_m R_0'$ is the more nearly will E approach twice E_1 .

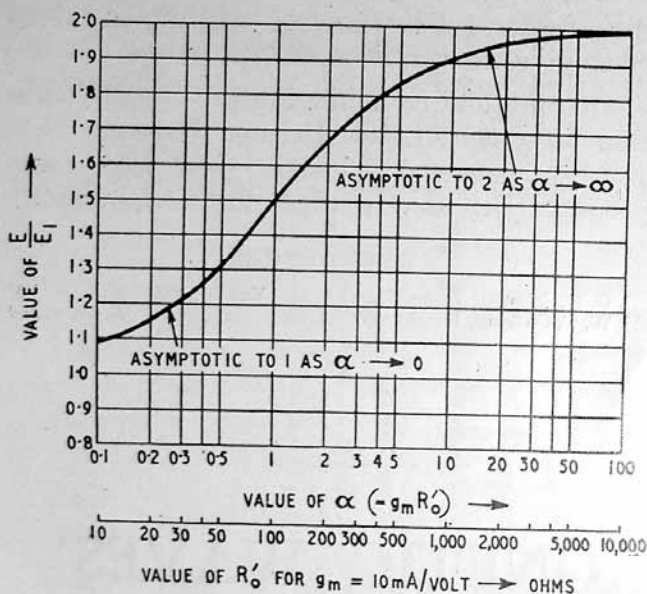


Fig. 3. Dependence of E/E_1 on the value of $g_m R_0'$.

in R_0' shall be a minimum and also for minimum value of total H.T. supply, the highest possible value of g_m is necessary. Also, since the input signal for the second valve is given by the product of R_0' and Δ (see equation (2)) it is a good thing to use valves requiring a small input signal, for these will require a smaller value of R_0' for a given Δ than other valves requiring bigger input signals. Pentodes or tetrodes, then, are more suitable to use in this particular circuit than triodes.

Input Signal Requirements

From (1) and (7), we have

$$E_1 = E - \frac{g_m R_0' E}{1 + 2g_m R_0'}$$

$$= E \left(\frac{1 + g_m R_0'}{1 + 2g_m R_0'} \right)$$

$$\therefore \frac{E}{E_1} = \frac{1 + 2g_m R_0'}{1 + g_m R_0'}$$

Now the fraction $\frac{E}{E_1}$ gives the value of the total input signal across AB (Fig. 1) over the input signal required by V_1 . Let

Dependence of Output Power on Value of $g_m R_0'$

It is also interesting to find the effect of varying R_0' on the power output of the two valves. Since i_1 exceeds i_2 , it follows that if V_1 is driven to the limit of its power output, then V_2 is being under-run to an extent governed by the value of R_0' . The power actually delivered by the two valves

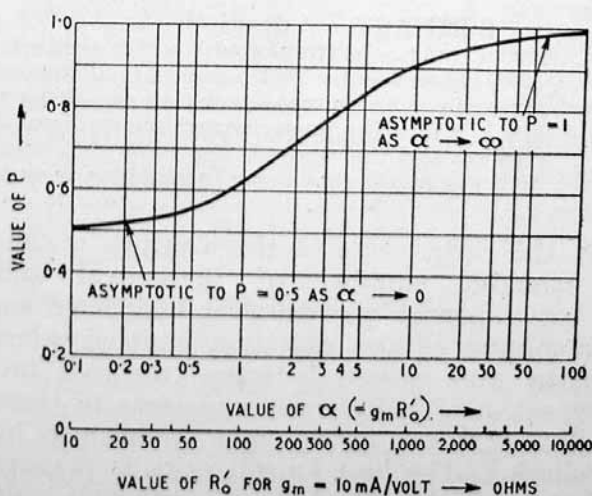


Fig. 4. Illustrating the dependence of output power ratio on the value of α .

is proportional to $i_1^2 + i_2^2$ whereas if both anode currents equalled i_1 the power would be proportional to $2i_1^2$. Hence the fraction of the maximum possible power output actually obtained, P , is defined by

$$P = \frac{i_1^2 + i_2^2}{2i_1^2}$$

$$= \frac{1}{2} \left(1 + \frac{i_2^2}{i_1^2} \right)$$

From (5) and (6)

$$\frac{i_2}{i_1} = \frac{g_m R_0'}{1 + g_m R_0'}$$

$$\therefore P = \frac{1}{2} \left[1 + \frac{g_m^2 R_0'^2}{(1 + g_m R_0')^2} \right]$$

$$= \frac{1}{2} \left[1 + \frac{\alpha^2}{(1 + \alpha)^2} \right]$$

where $\alpha = g_m R_0'$

In Fig. 4 this fraction has been plotted in the form of a curve and from this we can

estimate the value of α necessary to give the desired power output. Suppose, for example, we want at least 90 per cent. of the maximum possible power output. For this value $\alpha = 10$ from Fig. 4, so that, if $g_m = 10$ mA/volt for the particular valves used $R_0' = \frac{\alpha}{g_m} = \frac{10}{10} = 1,000$ ohms. Clearly

the greater g_m is the less will R_0' need to be to give the required output power.

This method of achieving push-pull operation is covered by British Patents Nos. 492,407 (General Electric Co., Ltd.) and 508,697 (British Broadcasting Corporation).

REFERENCES

- ¹ O. H. Schmitt, *Journ. Scient. Instr.*, March, 1938, Vol. 15, p. 100.
- ² W. T. Cocking, *Wireless World*, 13th April, 1939, Vol. 44, p. 340.